Direct Numerical Simulation of Fully Resolved Liquid Droplets in a Turbulent Flow

Michele Rosso & Said Elghobashi

Department of Mechanical and Aerospace Engineering
University of California, Irvine

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Objective & motivation

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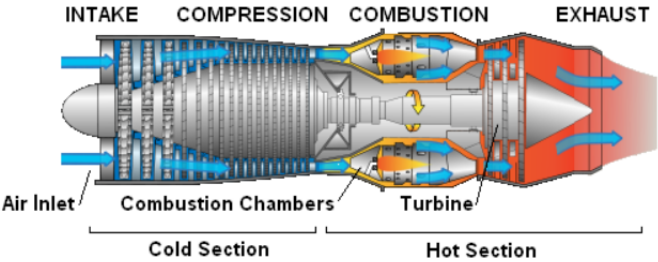
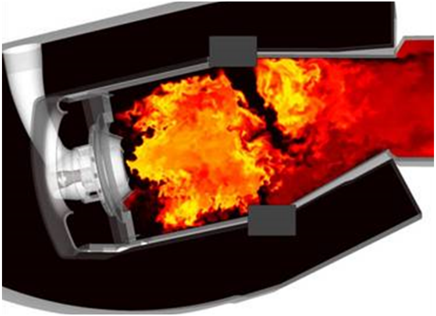
Motivation Dispersed liquid/gas multiphase flows occur in a wide range of natural phenomena and engineering devices, e.g. combustion of liquid fuel sprays.

Example of application I

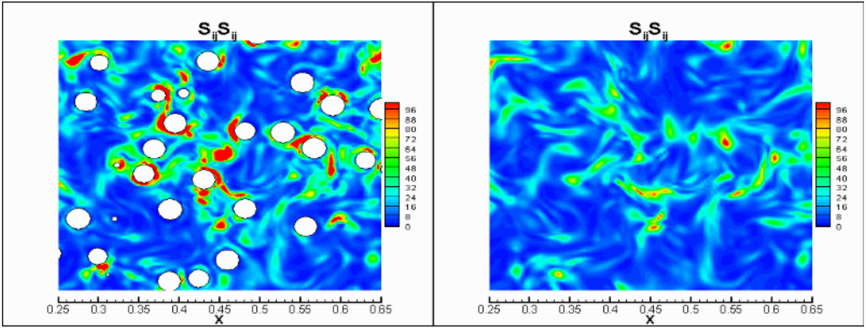


Example of application II



Effect of dispersed solid spherical particles on the dissipation rate of TKE (Lucci, Ferrante & Elghobashi, JFM 2010)

Re_λ	N	N_p	Φ_v	d/η
75	256	6400	0.1	16



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- On Blue Waters the simulation required 12 hrs and 65536 processors to advance the solution for about 3 large-eddy turnover times
- The simulation of dispersed two-phase turbulence requires about double the time of single-phase turbulence

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- Momentum equations:

$$\frac{\partial \mathbf{u}}{\partial t} + \nabla \cdot (\mathbf{uu}) = -\frac{1}{\rho}\left[\nabla p + \frac{\nabla \cdot \mathbf{T}}{Re} - \frac{\rho}{Fr}\mathbf{k} + \frac{\mathbf{f}_\sigma}{We}\right]$$

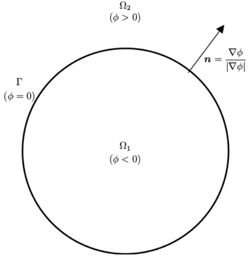
Dimensionless parameters:

$$Re = \frac{\widetilde{\rho}_{gas}\widetilde{U}\widetilde{L}}{\widetilde{\mu}_{gas}} \qquad Fr = \frac{\widetilde{U}^2}{\widetilde{g}\widetilde{L}} \qquad We = \frac{\widetilde{\rho}_{gas}\widetilde{U}^2\widetilde{L}}{\widetilde{\sigma}}$$

Interface definition & transport

The gas/liquid interface $\Gamma(t)$ is described as the zero level set of a signed distance function $\phi(\mathbf{x}, t)$ that is transported by the fluid velocity $\mathbf{u}$ via:

$$\frac{\partial \phi}{\partial t} + \mathbf{u} \cdot \nabla \phi = 0$$



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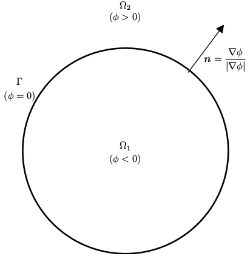

In order to keep ϕ a signed distance function, a reinitialization equation is solved until convergence:

$$\frac{\partial \phi}{\partial \tau} = \text{sign}(\phi_0)(1 - |\nabla \phi|)$$

where τ is a fictitious time and ϕ_0 is the level set function after the advection step.

Variable-Density Projection Method

1. A provisional velocity $\mathbf{u}^*$ is computed from $\mathbf{u}^n$, the velocity field at time n:

$$\frac{\mathbf{u}^* - \mathbf{u}^n}{\delta t} = \mathbf{R}^n$$

where $\mathbf{R}^n$ includes the convective, viscous, surface tension and gravity terms.

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3. $\mathbf{u}^*$ is corrected to obtain $\mathbf{u}^{n+1}$, the velocity field at time $n+1$:

$$\mathbf{u}^{n+1} = \mathbf{u}^* - \delta t \frac{\nabla p^{n+1}}{\rho^{n+1}}$$

Solution of the Variable-Coefficients Poisson's equation

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- A Geometric Algebraic Multigrid preconditioner is used
- We rely on the PETSc library for the solution of the linear system

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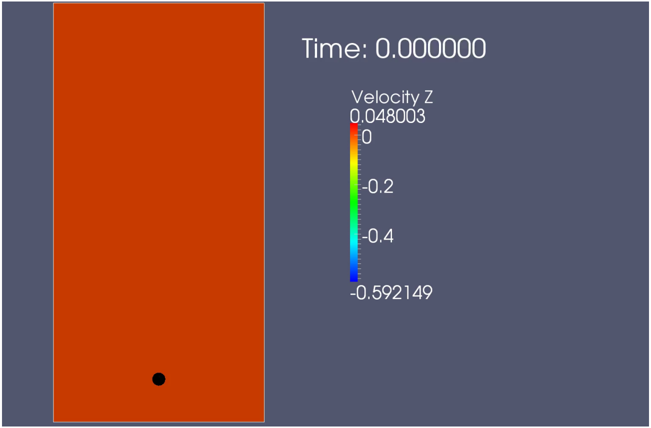
- Therefore there is a limit on the number of processors that can be used for a given grid size
- We will try different solutions, e.g. OpenMP/MPI, improve communication topology ...

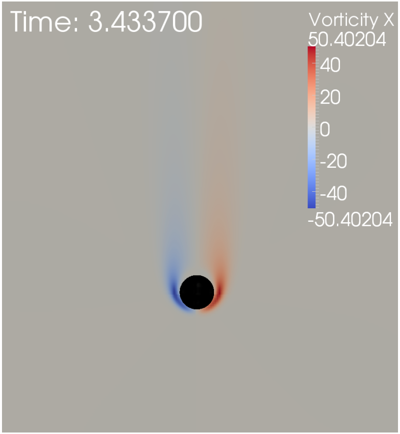
Falling droplet test I

Re_D	Fr_D	We_D	ρ_l/ρ_g	μ_l/μ_g
96	1009	0.06	829	54



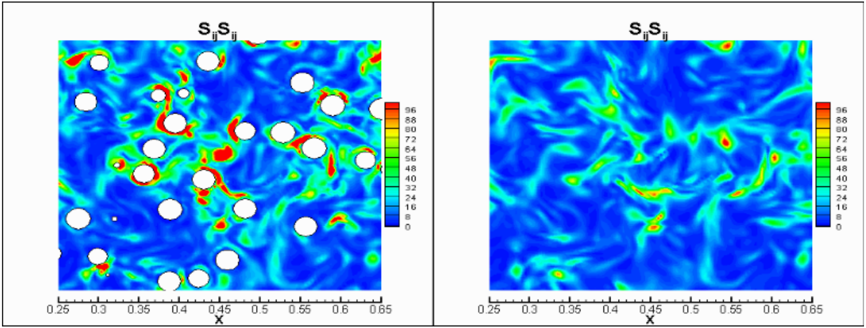
Falling droplet test II

VORTICITY ALONG X-AXIS



Future developments

We will replace the solid particles with droplets, and examine the effect on the turbulence structure.



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